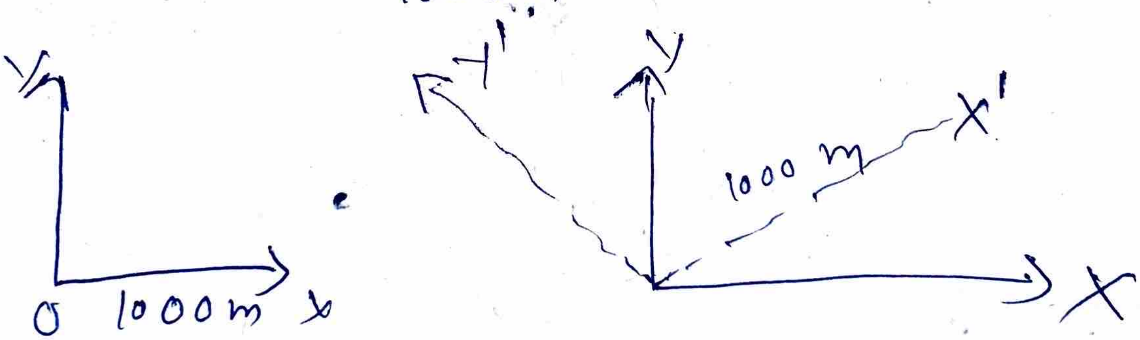
Transformation of Scalar under rotation

Scalar quantity → which has magnitude but no direction is called Scalar quantity

→ Consider a car moving on a road, the distance travelled by the car is Scalar quantity.

→ Car travels a distance of 1000m in xy plane if the plane is rotated about z-axis, i.e. X'-Y' plane, then the distance travelled by the car is 1000m

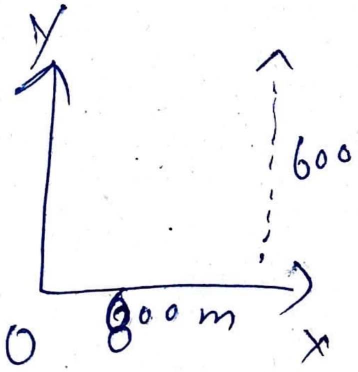


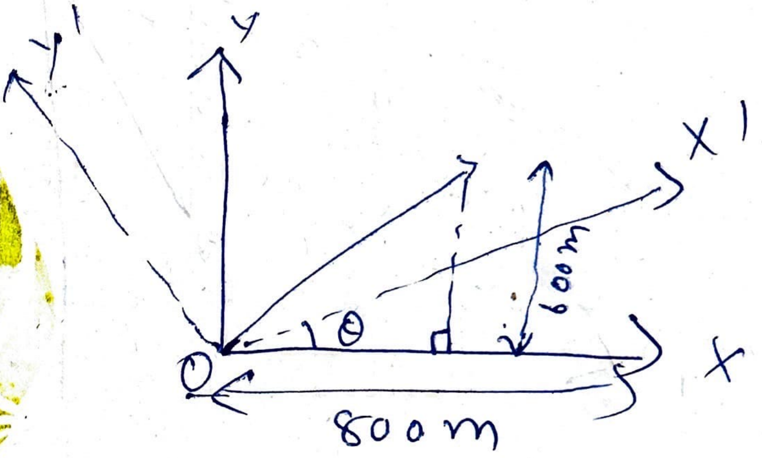
The distance travelled is same then it rotate at X-Y plane on X'-Y' plane. Hence, Scalar quantity remain unchanged.

Transformation of vector under Rotation

→ A quantity which has both magnitude & direction

→ Consider a car moving 800 m along X-axis & 600 m long Y-axis





Total distance travelled by Car is 1400m in XY plane

i.e. 800m + 600m = 1400m

The net displacement is vector 1000m in magnitude at an angle i.e. displacement

$$(hyp) = \sqrt{(600)^2 + (800)^2}$$

$$= 1000\,m$$

$$\text{angle} = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{6}{8}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

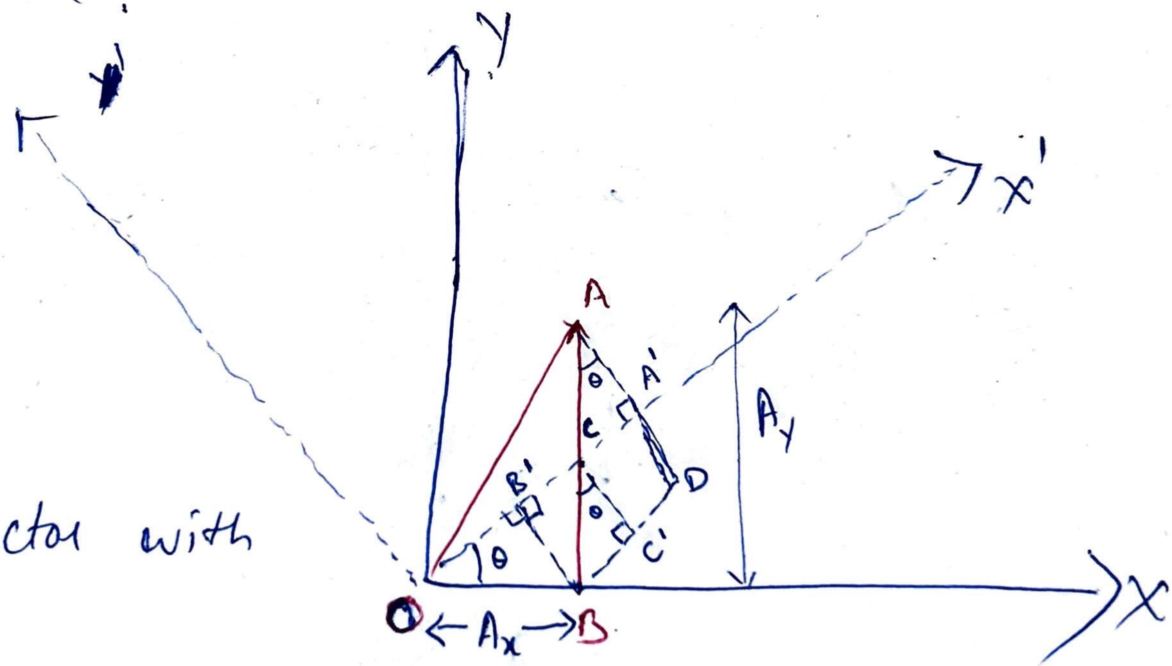
In X-Y plane, distance is unchanged.

→ It can be seen from figure (2) the vector Components $x'-y'$ are different

→ Hence, the vector Components are different in rotated plane.

→ Relation b/w Components of original & rotated frame obtained follows

→ Consider original plane $(x-y)$ & rotated plane (x^*-y')



OA is a vector with Components

$$A_x = OB$$
$$A_y = AB$$
$$A_x' = OA'$$
$$A_y' = AA'$$

Consider

$$A_x' = OA' = OB' + B'A'$$

$$\left[\begin{array}{l} OB = \frac{OB'}{\cos\theta} \\ OB' = OB\cos\theta \end{array}\right] \left[\begin{array}{l} \sin\theta = \frac{B'C}{BC} \\ B'C = BC\sin\theta \end{array}\right] \left[\begin{array}{l} \sin\theta = \frac{CA'}{CA} \\ CA' = CA\sin\theta \end{array}\right]$$

$$A_x' = OB\cos\theta + B'C + CA'$$

$$= OB\cos\theta + BC\sin\theta + CA\sin\theta$$

$$= OB\cos\theta + (BC + CA)\sin\theta \qquad \boxed{BC + CA = AB}$$

$$A_x' = OB\cos\theta + AB\sin\theta$$

$$\boxed{A_x' = A_x\cos\theta + A_y\sin\theta}$$

Similarly, component A_y' can be written as

$$A_y' = AA' \qquad \{AA' = AD - A'D\}$$

$$[A'D = BB']$$

$$= AD - A'D$$

$$= AB\cos\theta - OB\sin\theta \qquad \left\{\begin{array}{l} \cos\theta = \frac{AD}{AB} \\ AD = AB\cos\theta \end{array}\right\} \left[\begin{array}{l} \sin\theta = \frac{BB'}{OB} \\ BB' = OB\sin\theta \end{array}\right]$$

$$= A_y\cos\theta - A_x\sin\theta$$

$$A_y' = -A_x\sin\theta + A_y\cos\theta$$

if a vector is seen from a frame rotated about z-axis by angle.

-its x, y components in x'-y' frame

i.e $A_x' = A_x \cos\theta + A_y \sin\theta$

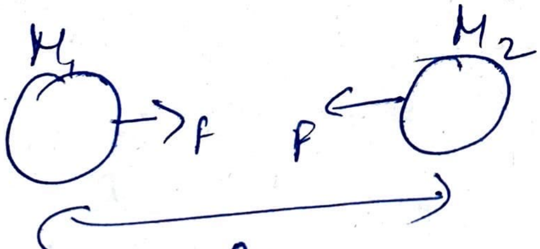
$A_y' = -A_x \sin\theta + A_y \cos\theta$

$A_z' = A_z$

Forces in nature :-

1) Gravitational force :- Mutual attraction b/w any 2-objects due to their masses

* It is universal force

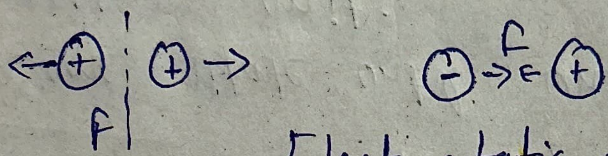


$$F = \frac{G M_1 M_2}{r^2}$$ universal gravitational law.

* High range
* No medium required

2). ~~Gravitational~~ Electromagnetic forces :-

⇒ Force b/w two charge bodies is E M Force.



Electrostatic force

* Moving charges
* Magnetic force due to magnetic field.
* High range
* No medium required

3) Strong Nuclear force

This force binds protons & neutrons in nucleus.

* It is charge Independent
* Only find inside nucleus
* Short range force
* This force doesn't act b/w e^- & e^-

Weak Nuclear force

→ This force appear in nuclear process such as β-decay of nucleus.

* β-decay of nucleus emit an electron & an uncharged particle called neutrino
* Range is very small (10^{-16} m)

Newton's Laws of motion

First Law :- If a body is in rest or in uniform motion it will remain same state until unless, when external force is applied on it.

Second Law : Force Applied is equal to rate of change of linear momentum of the body.

$$F \propto \frac{dp}{dt} = \frac{d(mv)}{dt} = ma$$

$$\vec{F} = \frac{dP}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

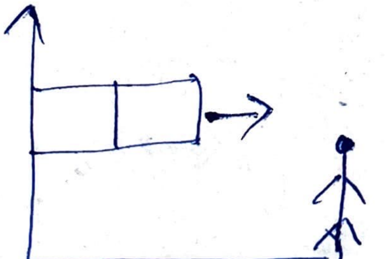
$m =$ Constant

$$\boxed{\vec{F} = m\vec{a}}$$

$V =$ Constant

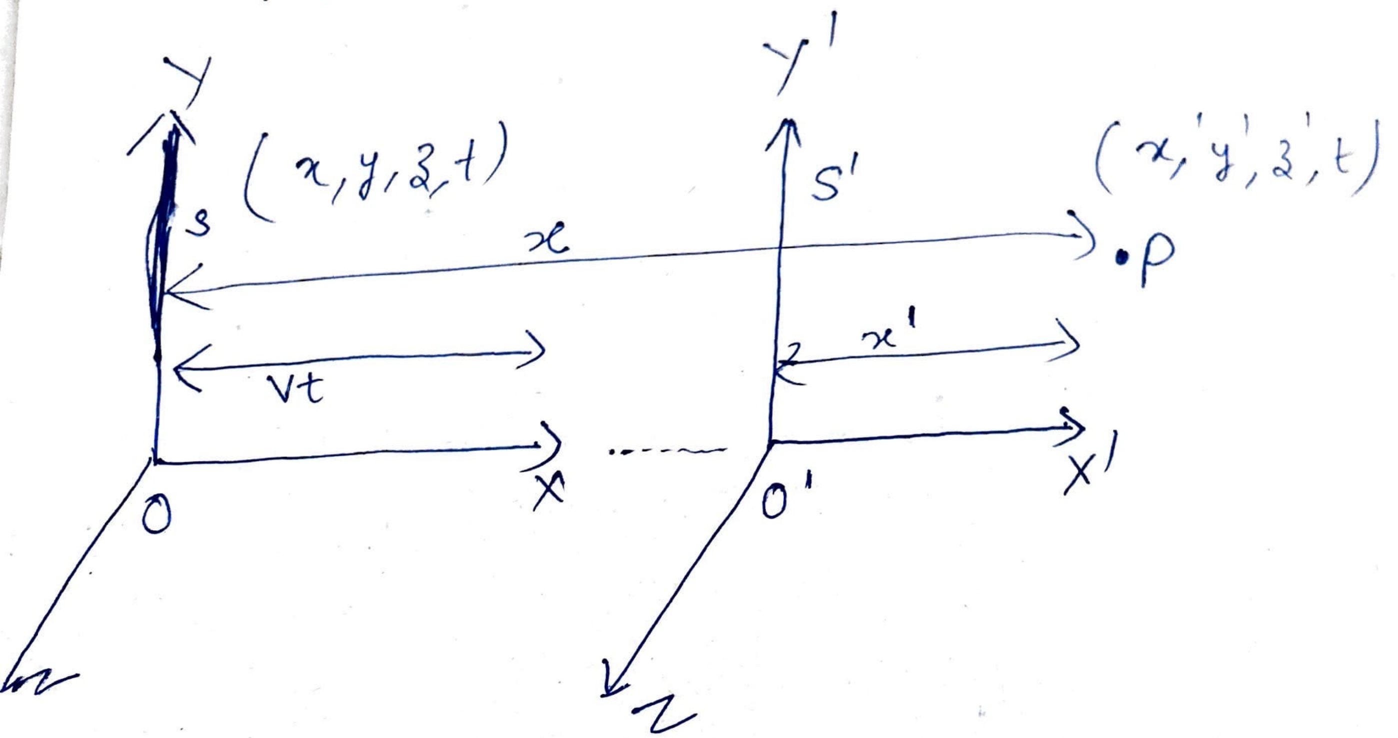
$$\vec{F} = \vec{U}\left(\frac{d\vec{m}}{dt}\right) \quad \left(\frac{dm}{dt} = \text{Mass flow rate}\right)$$

3. Each action has equal & opposite reaction but acting on different bodies.



Form Invariance of Newton's Second Law :-

Consider a set of two inertial frames (S & S') of references as shown in figure.



Invariant :- A quantity or law which does not change under a transformation (Gallilean transf…) is called as Invariant.

Eg:- Mass, length, time, Newton's second law

Explanation :-

* From figure, frame of reference 'S' is parallel to frame of reference S'.
* Coordinate axis x, y, z of reference S are parallel to axis.
* frame S move with a uniform velocity v along x-axis
* frame of reference Coincide at time $t = 0$
* Particle $P(x, y, z)$ is moving along the +ve x-axis is 't', within the first frame 'S'.
* The particle $P(x, y, z)$ is moving along the +ve axis is 't'. within the first frame 'S'.

* Thus, the Coordinates of $P'(x', y', z')$ can be transformed as i.e.

$$P(x, y, z) \rightarrow P'(x', y', z')$$

Transformed

$$x = x' + vt$$

$$x' = x - vt \quad \text{—(1)}$$

$$y' = y \quad \text{—(2)}$$

$$z' = z \quad \text{—(3)}$$

Gallilean transformation

The Coordinate y' & z' remain unchanged because no relative motion is observed along y' & z axis.

As the time is independent of space Coordinate System in classical physics

i.e. $t' = t$ —— (4)

Acc. to Newton's Second law of motion

$$F = ma$$

$$= m\frac{dv}{dt}$$

$$= m\frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$F = m\frac{d^2x}{dt^2} \quad \text{—— (5)}$$

Differentiating eqn (1) w.r.t 't'

Then $\frac{dx'}{dt} = \frac{dx}{dt} - \frac{d}{dt}(vt)$

$$\frac{dx'}{dt} = \frac{dx}{dt} - v\cancel{\frac{dt}{dt}}$$

$$\frac{dx'}{dt} = \frac{dx}{dt} - V$$

differentiating again $\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2} = 0 \quad (V = \text{Constant})$

$$\boxed{\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}} \longleftarrow (6)$$

Forces & Masses are absolute quantities in newton Mechanics

$M' = m \quad F' = F$

forces in fram S' is

from eqn (5) $F' = m' \frac{d^2x'}{dt^2}$ ——— (7)

force in frame 'S' is

$f = m \frac{d^2x}{dt^2}$ ——— (8)

But eqn (6) tells us

$$\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}.$$

eqn (8)

$$F = \frac{m' d^2x'}{dt^2} \quad \text{—} \quad (9)$$

From eqn (6), (7), (8), & (9)

$F = F'$

$$\boxed{\frac{m' dx'^2}{dt^2} = m \frac{dx^2}{dt^2}}$$

Second law is Invariant under ~~tra~~ transformation.